Interval neutrosophic sets and topology

Francisco Gallego Lupiáñez
Faculty of Mathematics, University Complutense, Madrid, Spain

Abstract

Purpose – In 2005, Smarandache generalized the Atanassov's intuitionistic fuzzy sets (IFSs) to neutrosophic sets (NS), and other researchers introduced the notion of interval neutrosophic set (INSs), which is an instance of NS, and studied various properties. The notion of neutrosophic topology on the non-standard interval is also due to Smarandache. The purpose of this paper is to study relations between INSs and topology.

Design/methodology/approach – The paper investigates the possible relations between INSs and topology.

Findings – Relations on INSs and neutrosophic topology.

Research limitations/implications – Clearly, the paper is confined to IFSs and NSs.

Practical implications – The main applications are in the mathematical field.

Originality/value – The paper shows original results on fuzzy sets and topology.

Keywords Set theory, Topology, Cybernetics, Fuzzy logic

Paper type Research paper

1. Introduction

In various recent papers, Smarandache (2002, 2003, 2005) generalizes intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs).

The notion of IFSs defined by Atanassov (1983, 1986) has been applied by Çoker (1997) for study intuitionistic fuzzy topological spaces (IFTS). This concept has been developed by many authors (Bayhan and Çoker, 2003; Çoker, 1996, 1997; Çoker and Eş, 1995; Eş and Çoker, 1996; Gürçay *et al.*, 1997; Hanafy, 2003; Hur *et al.*, 2004; Lee and Lee, 2000; Lupiáñez, 2004a, b, 2006a, b, 2007; Turanh and Çoker, 2000).

Smarandache also defined the notion of neutrosophic topology on the non-standard interval (Smarandache, 2002).

One can expect some relation between the intuitionistic fuzzy topology (IFT) on an IFS and the neutrosophic topology. We show in (Lupiáñez, 2008) that this is false. Indeed, an IFT is not necessarilly a neutrosophic topology.

Also (Wang *et al.*, 2005) introduced the notion of interval neutrosophic set (INSs), which is an instance of NS and studied various properties. We study in this paper relations between INSs and topology.

2. Basic definitions

First, we present some basic definitions. For definitions on non-standard analysis (Robinson, 1996):

Definition 1. Let X be a non-empty set. An IFS A, is an object having the form $A = \{ < x, \mu_A, \gamma_A > / x \in X\}$ where the functions $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$ (Atanassov, 1983).

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Definition 2. Let X be a non-empty set, and the IFSs $A = \{ < x, \mu_A, \gamma_A > |x \in X\}, B = \{ < x, \mu_B, \gamma_B > |x \in X\}$. Let:

- $\bar{A} = \{ < x, \gamma_A, \mu_A > |x \in X\}$;
- $A \cap B = \{ < x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B > |x \in X\}$; and
- $A \cup B = \{ < x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B > |x \in X\}$ (Atanassov, 1988).

Definition 3. Let X be a non-empty set. Let $0_\sim = \{ < x, 0, 1 > |x \in X\}$ and $1_\sim = \{ < x, 1, 0 > |x \in X\}$ (Çoker, 1997).

Definition 4. An IFT on a non-empty set X is a family τ of IFSs in X satisfying:

- $0_\sim, 1_\sim \in \tau$;
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$; and
- $\cup G_j \in \tau$ for any family $\{G_j | j \in J\} \subset \tau$.

In this case the pair (X, τ) is called an IFTS and any IFS in τ is called an intuitionistic fuzzy open set in X (Çoker, 1997).

Definition 5. Let T, I, F be real standard or non-standard subsets of the non-standard unit interval $]^-0, 1^+[$, with:

- $\sup T = t_{\sup}$, $\inf T = t_{\inf}$;
- $supI = i_{sup}$, $\inf I = i_{\inf}$; and
- $\sup F = f_{\sup}$, $\inf F = f_{\inf}$ and $n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup}$ $n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf}$.

T, I, F are called neutrosophic components. Let U be an universe of discourse, and M a set included in U. An element x from U is noted with respect to the set M as $x(T, I, F)$ and belongs to M in the following way: it is t% true in the set, i% indeterminate (unknown if it is) in the set, and f% false, where t varies in T, i varies in I, f varies in F. The set M is called a NS (Smarandache, 2005).

Remark. All IFS is a NS.

Definition 6. Let X be a space of points (objects) with generic elements in X denoted by x. An INS A in X is characterized by thuth-membership function T_A, indeteminacy-membership function I_A and falsity-membership function F_A. For each point x in X, we have that $T_A(x), I_A(x), F_A(x) \in [0, 1]$ (Wang *et al.*, 2005).

Remark. All INS is clearly a NS.

Definition 7.

- An INSs A is empty if $\inf T_A(x) = \sup T_A(x) = 0$, $\inf I_A(x) = \sup I_A(x) = 1$, $\inf F_A(x) = \sup F_A(x) = 0$ for all x in X.
- Let $\underline{0} = < 0, 1, 1 >$ and $\underline{1} = < 1, 0, 0 >$ (Wang *et al.*, 2005).

Definition 8. Let C_N denote a neutrosophic complement of A.

Then C_N is a function $C_N : N \to N$ and C_N must satisfy at least the following three axiomatic requirements:

(1) $C_N(\underline{0}) = \underline{1}$ and $C_N(\underline{1}) = \underline{0}$ (boundary conditions);

(2) let A and B be two INSs defined on X, if $A(x) \leq B(x)$, then $C_N(A(x)) \geq C_N(B(x))$, for all x in X (monotonicity); and

(3) let A be an INSs defined on X, then $C_N(C_N(A(x))) = A(x)$, for all x in X (involutivity) (Wang *et al.*, 2005).

Definition 9. Let I_N denote a neutrosophic intersection of two INSs A and B. Then I_N is a function $I_N : N \times N \rightarrow N$ and I_N must satisfy at least the following four axiomatic requirements:

(1) $I_N(A(x), \underline{1}) = A(x)$, for all x in X (boundary condition).

(2) $B(x) \leq C(x)$ implies $I_N(A(x), B(x)) \leq I_N(A(x), C(x))$, for all x in X (monotonicity).

(3) $I_N(A(x), B(x)) = I_N(B(x), A(x))$, for all x in X (commutativity).

(4) $I_N(A(x), I_N(B(x), C(x))) = I_N(I_N(A(x), B(x)), C(x))$, for all x in X (associativity) (Wang *et al.*, 2005).

Definition 10. Let U_N denote a neutrosophic union of two INSs A and B. Then U_N is a function $U_N : N \times N \rightarrow N$ and U_N must satisfy at least the following four axiomatic requirements:

(1) $U_N(A(x), \underline{0}) = A(x)$, for all x in X (boundary condition).

(2) $B(x) \leq C(x)$ implies $U_N(A(x), B(x)) \leq U_N(A(x), C(x))$, for all x in X (monotonicity).

(3) $U_N(A(x), B(x)) = U_N(B(x), A(x))$, for all x in X (commutativity).

(4) $U_N(A(x), U_N(B(x), C(x))) = U_N(U_N(A(x), B(x)), C(x))$, for all x in X (associativity) (Wang *et al.*, 2005).

3. Results

Proposition 1. Let A be an IFS in X, and $j(A)$ be the corresponding INS. We have that the complement of $j(A)$ is not necessarily $j(\bar{A})$.

Proof. If $A =< x, \mu_A, \gamma_A >$ is $j(A) =< \mu_A, 0, \gamma_A >$.

Then:

- for $0_\sim =< x, 0, 1 >$ is $j(0_\sim) = j(< x, 0, 1 >) =< 0, 0, 1 > \neq \underline{0} =< 0, 1, 1 >$; and
- for $1_\sim =< x, 1, 0 >$ is $j(1_\sim) = j(< x, 1, 0 >) =< 1, 0, 0 >= \underline{1}$

Thus, $1_\sim = \bar{0}_\sim$ and $j(1_\sim) = \underline{1} \neq C_N(j(\bar{0}_\sim))$ because $C_N(\underline{1}) = \underline{0} \neq j(\bar{0}_\sim)$. □

Definition 11. Let us construct a neutrosophic topology on $NT =]^-0, 1^+[$, considering the associated family of standard or non-standard subsets included in NT, and the empty set which is closed under set union and finite intersection neutrosophic. The interval NT endowed with this topology forms a neutrosophic topological space (Smarandache, 2002).

Proposition 2. Let (X, τ) be an IFTS. Then, the family of INSs $\{j(U)|U \in \tau\}$ is not necessarily a neutrosophic topology.

Proof. Let $\tau = \{1_\sim, 0_\sim, A\}$ where $A =< x, 1/2, 1/2 >$ then $j(1_\sim) = \underline{1}$, $j(0_\sim) =< 0, 0, 1 > \neq \emptyset$ and $j(A) =< 1/2, 0, 1/2 >$. Thus, $\{j(1_\sim), j(0_\sim), j(A)\}$ is not a neutrosophic topology, because the empty INS is not in this family. □

References

Atanassov, K.T. (1983), "Intuitionistic fuzzy sets", paper presented at the VII ITKR's Session, Sofia, June.

Atanassov, K.T. (1986), "Intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, Vol. 20, pp. 87-96.

Atanassov, K.T. (1988), "Review and new results on intuitionistic fuzzy sets", preprint IM-MFAIS-1-88, Sofia.

Bayhan, S. and Çoker, D. (2003), "On T_1 and T_2 separation axioms in intuitionistic fuzzy topological spaces", *J. Fuzzy Math.*, Vol. 11, pp. 581-92.

Çoker, D. (1996), "An introduction to fuzzy subspaces in intuitionistic fuzzy topological spaces", *J. Fuzzy Math*, Vol. 4, pp. 749-64.

Çoker, D. (1997), "An introduction to intuitionistic fuzzy topologial spaces", *Fuzzy Sets and Systems*, Vol. 88, pp. 81-9.

Çoker, D. and Eş, A.H. (1995), "On fuzzy compactness in intuitionistic fuzzy topological spaces", *J. Fuzzy Math.*, Vol. 3, pp. 899-909.

Eş, A.H. and Çoker, D. (1996), "More on fuzzy compactness in intuitionistic fuzzy topological spaces", *Notes IFS*, Vol. 2 No. 1, pp. 4-10.

Gürçay, H., Çoker, D. and Eş, A.H. (1997), "On fuzzy continuity in intuitionistic fuzzy topological spaces", *J. Fuzzy Math.*, Vol. 5, pp. 365-78.

Hanafy, J.H. (2003), "Completely continuous functions in intuitionistic fuzzy topological spaces", *Czech. Math. J.*, Vol. 53 No. 128, pp. 793-803.

Hur, K., Kim, J.H. and Ryou, J.H. (2004), "Intuitionistic fuzzy topologial spaces", *J. Korea Soc. Math. Educ., Ser B*, Vol. 11, pp. 243-65.

Lee, S.J. and Lee, E.P. (2000), "The category of intuitionistic fuzzy topological spaces", *Bull. Korean Math. Soc.*, Vol. 37, pp. 63-76.

Lupiáñez, F.G. (2004a), "Hausdorffness in intuitionistic fuzzy topological spaces", *J. Fuzzy Math.*, Vol. 12, pp. 521-5.

Lupiáñez, F.G. (2004b), "Separation in intuitionistic fuzzy topological spaces", *Intern. J. Pure Appl. Math.*, Vol. 17, pp. 29-34.

Lupiáñez, F.G. (2006a), "Nets and filters in intuitionistic fuzzy topological spaces", *Inform. Sci.*, Vol. 176, pp. 2396-404.

Lupiáñez, F.G. (2006b), "On intuitionistic fuzzy topological spaces", *Kybernetes*, Vol. 35, pp. 743-7.

Lupiáñez, F.G. (2007), "Covering properties in intuitionistic fuzzy topological spaces", *Kybernetes*, Vol. 36, pp. 749-53.

Lupiáñez, F.G. (2008), "On neutrosophic topology", *Kybernetes*, Vol. 37 No. 6, pp. 797-800.

Robinson, A. (1996), *Non-standard Analysis*, Princeton University Press, Princeton, NJ.

Smarandache, F. (2002), "A unifying field in logics: neutrosophic logic", *Multiple-valued Logic*, Vol. 8, pp. 385-438.

Smarandache, F. (2003), "Definition of neutrosophic logic. A generalization of the intuitionistic fuzzy logic", *Proc. 3rd Conf. Eur. Soc. Fuzzy Logic Tech. [EUSFLAT, 2003]*, pp. 141-6.

Smarandache, F. (2005), "Neutrosophic set. A generalization of the intuitionistic fuzzy set", *Intern. J. Pure Appl. Math.*, Vol. 24, pp. 287-97.

Turanh, N. and Çoker, D. (2000), "Fuzzy connectedness in intuitionistic fuzzy topological spaces", *Fuzzy Sets and Systems*, Vol. 116, pp. 369-75.

Wang, H., Smarandache, F., Zhang, Y.-Q. and Sunderraman, R. (2005), *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*, Hexis, Phoenix, AZ.

Corresponding author
Francisco Gallego Lupiáñez can be contacted at: fg_lupianez@mat.ucm.es